# Multiplication and Division of Whole Numbers: Division of Whole Numbers* 

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#### Abstract

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to divide whole numbers. By the end of the module students should be able to be able to divide a whole number by a single or multiple digit divisor and interpret a calculator statement that a division results in a remainder.


## 1 Section Overview

- Division with a Single Digit Divisor
- Division with a Multiple Digit Divisor
- Division with a Remainder
- Calculators


## 2 Division with a Single Digit Divisor

Our experience with multiplication of whole numbers allows us to perform such divisions as $75 \div 5$. We perform the division by performing the corresponding multiplication, $5 \times Q=75$. Each division we considered in here ${ }^{1}$ had a one-digit quotient. Now we will consider divisions in which the quotient may consist of two or more digits. For example, $75 \div 5$.

Let's examine the division $75 \div 5$. We are asked to determine how many 5 's are contained in 75 . We'll approach the problem in the following way.

1. Make an educated guess based on experience with multiplication.
2. Find how close the estimate is by multiplying the estimate by 5 .
3. If the product obtained in step 2 is less than 75 , find out how much less by subtracting it from 75 .

[^0]4. If the product obtained in step 2 is greater than 75 , decrease the estimate until the product is less than 75 . Decreasing the estimate makes sense because we do not wish to exceed 75 .

We can suggest from this discussion that the process of division consists of

## The Four Steps in Division

1. an educated guess
2. a multiplication
3. a subtraction
4. bringing down the next digit (if necessary)

The educated guess can be made by determining how many times the divisor is contained in the dividend by using only one or two digits of the dividend.

### 2.1 Sample Set A

## Example 1

Find $75 \div 5$.
$5 \overline{75}$ Rewrite the problem using a division bracket.
10
$5 \overline{75}$
Make an educated guess by noting that one 5 is contained in 75 at most 10 times.
Since 7 is the tens digit, we estimate that 5 goes into 75 at most 10 times.
10
$5 \longdiv { 7 5 }$
$-50$
25
Now determine how close the estimate is.
10 fives is $10 \times 5=50$. Subtract 50 from 75 .
Estimate the number of 5 's in 25 .
There are exactly 5 fives in 25 .
$5 \quad 10$ fives +5 fives $=15$ fives.
10 There are 15 fives contained in 75 .
$5 \overline{75}$
$-50$
25
$-25$
0
$75 \stackrel{15}{ } \times 5$
Check: $75 \cong 75$
Thus, $75 \div 5=15$.
The notation in this division can be shortened by writing.

Divide: 5 goes into 7 at most 1 time. Divide: 5 goes into 25 exactly 5 times.
\{ Multiply: $1 \times 5=5$. Write 5 below 7. \{ Multiply: $5 \times 5=25$. Write 25 below 25.
Subtract: 7-5=2. Bring down the 5. Subtract: $\quad 25-25=0$.

## Example 2

Find $4,944 \div 8$.
8) $\overline{4944}$

Rewrite the problem using a division bracket.

8) | 600 |
| ---: |
| 4944 |

$-4800$
144
8 goes into 49 at most 6 times, and 9 is in the hundreds column. We'll guess 600.
Then, $8 \times 600=4800$.
10
600
8) $\overline{4944}$
$-4800$
144

| $-\quad 80$ |
| :--- |

64
8 goes into 14 at most 1 time, and 4 is in the tens column. We'll guess 10 .
8
10
600
8) $\overline{4944}$
$-4800$
144
$-\quad 80$
64
$\underline{-64}$
0
8 goes into 64 exactly 8 times.

600 eights +10 eights +8 eights $=618$ eights.
$4944 \stackrel{?}{=} \times 618$
Check: $4944 \underline{\underline{1}} \mathbf{4 9 4 4}$
Thus, $4,944 \div 8=618$.
As in the first problem, the notation in this division can be shortened by eliminating the subtraction signs and the zeros in each educated guess.
$8 \longdiv { 4 9 4 4 }$
48 $\downarrow$
14

64
64
0
Divide: 8 goes into 49 at most 6 times. Divide: 8 goes into 14 at most 1 time. Divide: 8 goes into 6
$\{$ Multiply: $6 \times 8=48$. Write 48 below 49. \{ Multiply: $1 \times 8=8$. Write 8 below $14 . \quad\{$ Multiply: $8 \times 8=64$.
Subtract: $49-48=1$. Bring down the 4 . Subtract: $14-8=6$. Bring down the 4 . Subtract:

NOTE: Not all divisions end in zero. We will examine such divisions in a subsequent subsection.

### 2.2 Practice Set A

Perform the following divisions.
Exercise 1
(Solution on p. 15.)
$126 \div 7$
Exercise 2
(Solution on p. 15.)
$324 \div 4$
Exercise 3
(Solution on p. 15.)
$2,559 \div 3$
Exercise 4
(Solution on p. 15.)
$5,645 \div 5$
Exercise 5
(Solution on p. 15.) $757,125 \div 9$

## 3 Division with a Multiple Digit Divisor

The process of division also works when the divisor consists of two or more digits. We now make educated guesses using the first digit of the divisor and one or two digits of the dividend.

### 3.1 Sample Set B

## Example 3

Find $2,232 \div 36$.
$3 6 \longdiv { 2 2 3 2 }$
Use the first digit of the divisor and the first two digits of the dividend to make the educated guess.

3 goes into 22 at most 7 times.
Try 7: $7 \times 36=252$ which is greater than 223 . Reduce the estimate.
Try 6: $6 \times 36=216$ which is less than 223 .
$3 6 \longdiv { 2 2 3 2 }$
$-216 \downarrow$
72
Multiply: $6 \times 36=216$. Write 216 below 223.
Subtract: $\quad 223-216=7$. Bring down the 2 .
Divide 3 into 7 to estimate the number of times 36 goes into 72 . The 3 goes into 7 at most 2 times.

Try $2: 2 \times 36=72$.
62
$3 6 \longdiv { 2 2 3 2 }$
$\frac{216 \downarrow}{72}$
-72
0
$2232 \stackrel{?}{=} 36 \times 62$
Check: $2232 \underline{\underline{1} 2232}$
Thus, $2,232 \div 36=62$.

## Example 4

Find $2,417,228 \div 802$.

$$
8 0 2 \longdiv { 2 4 1 7 2 2 8 }
$$

First, the educated guess: $24 \div 8=3$. Then $3 \times 802=2406$, which is less than 2417 . Use 3 as the guess. Since $3 \times 802=2406$, and 2406 has four digits, place the 3 above the fourth digit of the dividend.

802) | $\frac{3}{2417228}$ |
| :---: |
| $-2406 \downarrow$ |
| 112 |

Subtract: $2417-2406=11$.
Bring down the 2.
The divisor 802 goes into 112 at most 0 times. Use 0 .

| 302 |
| ---: |
|  |
| $-\underline{2406 \downarrow}$ |
| 112 |
| $\frac{-0 \downarrow}{1122}$ |

$$
\begin{array}{lc}
\text { Multiply: } & 0 \times 802=0 \\
\text { Subtract: } & 112-0=112
\end{array}
$$

Bring down the 2 .
The 8 goes into 11 at most 1 time, and $1 \times 802=802$, which is less than 1122 . Try 1 .

| 301 |
| ---: |
| 2417228 |
| $-2406 \downarrow$ |
| $112 \downarrow$ |
| $\frac{-0}{1122}$ |
| $\frac{-802 \downarrow}{3208}$ |

Subtract $1122-802=320$
Bring down the 8 .
8 goes into 32 at most 4 times.
$4 \times 802=3208$.
Use 4.
3014
$8 0 2 \longdiv { 2 4 1 7 2 2 8 }$
$-2406 \downarrow$
112
$\frac{-0 \downarrow}{1122}$
$-802 \downarrow$
3208
$-\frac{3208}{0}$
$2417228 \stackrel{?}{=} 3014 \times 802$
Check: $2417228 \cong 2417228$
Thus, $2,417,228 \div 802=3,014$.

### 3.2 Practice Set B

Perform the following divisions.
Exercise 6
(Solution on p. 15.)
$1,376 \div 32$
Exercise 7
(Solution on p. 15.)
$6,160 \div 55$

| Exercise 8 | (Solution on p. 15.) |
| :--- | :--- |
| $18,605 \div 61$ |  |
| Exercise 9 | (Solution on p. 15.) |
| $144,768 \div 48$ |  |

## 4 Division with a Remainder

We might wonder how many times 4 is contained in 10 . Repeated subtraction yields 10
$\qquad$
6
$-4$
2
Since the remainder is less than 4 , we stop the subtraction. Thus, 4 goes into 10 two times with 2 remaining. We can write this as a division as follows.

$$
\begin{array}{r}
2 \\
4 \longdiv { 1 0 } \\
-\quad 8 \\
\hline 2
\end{array}
$$

Divide: 4 goes into 10 at most 2 times.
Multiply: $2 \times 4=8$. Write 8 below 0 .
Subtract: $\quad 10-8=2$.
Since 4 does not divide into 2 (the remainder is less than the divisor) and there are no digits to bring down to continue the process, we are done. We write

2 R2
4) 10
or $10 \div 4=\underbrace{2 R 2}$
$-8 \quad 2$ with remainder 2
2

### 4.1 Sample Set C

## Example 5

Find $85 \div 3$.
28
$3 \longdiv { 8 5 }$
$\frac{6 \downarrow}{25}$
$\frac{24}{1}$

Divide: 3 goes into 8 at most 2 times. Divide: 3 goes into 25 at most 8 times.
$\{$ Multiply: $2 \times 3=6$. Write 6 below 8 . \{ Multiply: $3 \times 8=24$. Write 24 below 25.
Subtract: $8-6=2$. Bring down the 5 . Subtract: $25-24=1$.
There are no more digits to bring down to continue the process. We are done. One is the remainder.

Check: Multiply 28 and 3 , then add 1.

$$
28
$$

$\begin{array}{r} \\ \times \quad 3 \\ \hline\end{array}$
84

| $+\quad 1$ |
| :--- | 85

Thus, $85 \div 3=28$ R 1 .

## Example 6

Find $726 \div 23$.
31
23726
$\frac{69 \downarrow}{36}$
$\frac{23}{13}$

Check: Multiply 31 by 23 , then add 13 .
31
$\begin{array}{r} \\ \times \quad 23 \\ \hline 93\end{array}$
93
62
$\overline{713}$
$\begin{array}{r}+\quad 13 \\ \hline 726\end{array}$
Thus, $726 \div 23=31 R 13$.

### 4.2 Practice Set C

Perform the following divisions.
Exercise 10
(Solution on p. 15.)
$75 \div 4$
Exercise 11
(Solution on p. 15.)
$346 \div 8$
Exercise 12
(Solution on p. 15.)
$489 \div 21$
Exercise 13
$5,016 \div 82$

## Exercise 14

(Solution on p. 15.)
$41,196 \div 67$

## 5 Calculators

The calculator can be useful for finding quotients with single and multiple digit divisors. If, however, the division should result in a remainder, the calculator is unable to provide us with the particular value of the remainder. Also, some calculators (most nonscientific) are unable to perform divisions in which one of the numbers has more than eight digits.

### 5.1 Sample Set D

Use a calculator to perform each division.

## Example 7

 $328 \div 8$| Type | 328 |
| :---: | :--- |
| Press | $\div$ |
| Type | 8 |
| Press | $=$ |

Table 1
The display now reads 41.

## Example 8

$53,136 \div 82$

| Type | 53136 |
| :--- | :--- |
| Press | $\div$ |
| Type | 82 |
| Press | $=$ |

Table 2
The display now reads 648 .

## Example 9

$730,019,001 \div 326$
We first try to enter $730,019,001$ but find that we can only enter 73001900 . If our calculator has only an eight-digit display (as most nonscientific calculators do), we will be unable to use the calculator to perform this division.
Example 10
$3727 \div 49$

| Type | 3727 |
| :---: | :--- |
| Press | $\div$ |
| Type | 49 |
| Press | $=$ |

Table 3
The display now reads 76.061224 .
This number is an example of a decimal number (see here ${ }^{2}$ ). When a decimal number results in a calculator division, we can conclude that the division produces a remainder.

### 5.2 Practice Set D

Use a calculator to perform each division.
Exercise 15
(Solution on p. 15.)
$3,330 \div 74$
Exercise 16
(Solution on p. 15.)
$63,365 \div 115$
Exercise 17
(Solution on p. 15.)
$21,996,385,287 \div 53$
Exercise 18
(Solution on p. 15.)
$4,558 \div 67$

## 6 Exercises

For the following problems, perform the divisions.
The first 38 problems can be checked with a calculator by multiplying the divisor and quotient then adding the remainder.

```
Exercise 19
                                    (Solution on p. 15.)
    52\div4
Exercise 20
    776 \div8
Exercise 21
                                    (Solution on p. 15.)
    603\div9
Exercise 22
    240\div8
Exercise 23
        208\div4
Exercise 24
        576\div6
Exercise 25
    (Solution on p. 15.)
```

        \(21 \div 7\)
    2"Decimals: Objectives" <http://cnx.org/content/m18894/latest/>
    Exercise 26

$$
0 \div 0
$$

Exercise 27
(Solution on p. 15.)
$140 \div 2$
Exercise 28 $528 \div 8$
Exercise 29 $244 \div 4$

Exercise 30

$$
0 \div 7
$$

Exercise 31 $177 \div 3$
(Solution on p. 16.)

Exercise 32 $96 \div 8$
Exercise 33 (Solution on p. 16.) $67 \div 1$
Exercise 34 $896 \div 56$
Exercise 35 $1,044 \div 12$
Exercise 36 $988 \div 19$
Exercise 37
(Solution on p. 16.) $5,238 \div 97$
Exercise 38 $2,530 \div 55$
Exercise 39 $4,264 \div 82$
Exercise 40 $637 \div 13$
Exercise 41 $3,420 \div 90$
Exercise 42

$$
5,655 \div 87
$$

Exercise 43
(Solution on p. 16.)

$$
2,115 \div 47
$$

Exercise 44 $9,328 \div 22$
Exercise 45 $55,167 \div 71$

Exercise 46 $68,356 \div 92$
Exercise 47 $27,702 \div 81$

Exercise 48
$6,510 \div 31$
Exercise 49
(Solution on p. 16.)
$60,536 \div 94$
Exercise 50
$31,844 \div 38$
Exercise 51
(Solution on p. 16.)
$23,985 \div 45$
Exercise 52 $60,606 \div 74$
Exercise 53
(Solution on p. 16.)

$$
2,975,400 \div 285
$$

Exercise 54

$$
1,389,660 \div 795
$$

Exercise 55
(Solution on p. 16.)

$$
7,162,060 \div 879
$$

Exercise 56

$$
7,561,060 \div 909
$$

Exercise 57 $38 \div 9$
Exercise 58

$$
97 \div 4
$$

## Exercise 59

(Solution on p. 16.)

$$
199 \div 3
$$

Exercise 60 $573 \div 6$
Exercise 61 $10,701 \div 13$
Exercise 62 $13,521 \div 53$
Exercise 63
$3,628 \div 90$
Exercise 64 $10,592 \div 43$
Exercise 65 $19,965 \div 30$
Exercise 66 $8,320 \div 21$
Exercise 67 $61,282 \div 64$
Exercise 68 $1,030 \div 28$
Exercise 69
(Solution on p. 16.)
$7,319 \div 11$

## Exercise 70

$3,628 \div 90$
Exercise 71
(Solution on p. 16.)
$35,279 \div 77$
Exercise 72
$52,196 \div 55$
Exercise 73
(Solution on p. 16.)

$$
67,751 \div 68
$$

For the following 5 problems, use a calculator to find the quotients.
Exercise $\mathbf{7 4}$
$4,346 \div 53$
Exercise 75
(Solution on p. 16.)
$3,234 \div 77$
Exercise 76
$6,771 \div 37$
Exercise 77
(Solution on p. 16.)
$4,272,320 \div 520$

## Exercise 78

$7,558,110 \div 651$
Exercise 79
(Solution on p. 16.)
A mathematics instructor at a high school is paid $\$ 17,775$ for 9 months. How much money does this instructor make each month?

## Exercise 80

A couple pays $\$ 4,380$ a year for a one-bedroom apartment. How much does this couple pay each month for this apartment?

## Exercise 81

(Solution on p. 17.)
Thirty-six people invest a total of $\$ 17,460$ in a particular stock. If they each invested the same amount, how much did each person invest?

## Exercise 82

Each of the 28 students in a mathematics class buys a textbook. If the bookstore sells $\$ 644$ worth of books, what is the price of each book?

## Exercise 83

(Solution on p. 17.)
A certain brand of refrigerator has an automatic ice cube maker that makes 336 ice cubes in one day. If the ice machine makes ice cubes at a constant rate, how many ice cubes does it make each hour?

## Exercise 84

A beer manufacturer bottles 52,380 ounces of beer each hour. If each bottle contains the same number of ounces of beer, and the manufacturer fills 4,365 bottles per hour, how many ounces of beer does each bottle contain?

## Exercise 85

(Solution on p. 17.)
A computer program consists of 68,112 bits. 68,112 bits equals 8,514 bytes. How many bits in one byte?

## Exercise 86

A 26-story building in San Francisco has a total of 416 offices. If each floor has the same number of offices, how many floors does this building have?

## Exercise 87

(Solution on p. 17.)
A college has 67 classrooms and a total of 2,546 desks. How many desks are in each classroom if each classroom has the same number of desks?

### 6.1 Exercises for Review

## Exercise 88

( here ${ }^{3}$ ) What is the value of 4 in the number 124,621 ?
Exercise 89
(Solution on p. 17.)
( here $^{4}$ ) Round 604,092 to the nearest hundred thousand.

## Exercise 90

( here ${ }^{5}$ ) What whole number is the additive identity?
Exercise 91
(Solution on p. 17.)
( here ${ }^{6}$ ) Find the product. 6, $256 \times 100$.

## Exercise 92

( here ${ }^{7}$ ) Find the quotient. $0 \div 11$.

[^1]http://cnx.org/content/m34865/1.2/

## Solutions to Exercises in this Module

Solution to Exercise 1 (p. 4)
18
Solution to Exercise 2 (p. 4)
81
Solution to Exercise 3 (p. 4)
853
Solution to Exercise 4 (p. 4)
1,129
Solution to Exercise 5 (p. 4)
84,125
Solution to Exercise 6 (p. 6)
43
Solution to Exercise 7 (p. 6)
112
Solution to Exercise 8 (p. 7)
305
Solution to Exercise 9 (p. 7)
3,016
Solution to Exercise 10 (p. 8) 18 R3
Solution to Exercise 11 (p. 8) 43 R2
Solution to Exercise 12 (p. 8) 23 R6
Solution to Exercise 13 (p. 8) 61 R14
Solution to Exercise 14 (p. 9) 614 R58
Solution to Exercise 15 (p. 10)
45
Solution to Exercise 16 (p. 10)
551
Solution to Exercise 17 (p. 10)
Since the dividend has more than eight digits, this division cannot be performed on most nonscientific calculators. On others, the answer is 415,026,137.4
Solution to Exercise 18 (p. 10)
This division results in 68.02985075 , a decimal number, and therefore, we cannot, at this time, find the value of the remainder. Later, we will discuss decimal numbers.
Solution to Exercise 19 (p. 10)
13
Solution to Exercise 21 (p. 10)
67
Solution to Exercise 23 (p. 10)
52
Solution to Exercise 25 (p. 10) 3
Solution to Exercise 27 (p. 11)
70

Solution to Exercise 29 (p. 11)
61
Solution to Exercise 31 (p. 11) 59
Solution to Exercise 33 (p. 11)
67
Solution to Exercise 35 (p. 11) 87
Solution to Exercise 37 (p. 11)
54
Solution to Exercise 39 (p. 11)
52
Solution to Exercise 41 (p. 11)
38
Solution to Exercise 43 (p. 11) 45
Solution to Exercise 45 (p. 11) 777
Solution to Exercise 47 (p. 11) 342
Solution to Exercise 49 (p. 12) 644
Solution to Exercise 51 (p. 12)
533
Solution to Exercise 53 (p. 12)
10,440
Solution to Exercise 55 (p. 12)
8,147 remainder 847
Solution to Exercise 57 (p. 12)
4 remainder 2
Solution to Exercise 59 (p. 12)
66 remainder 1
Solution to Exercise 61 (p. 12)
823 remainder 2
Solution to Exercise 63 (p. 12)
40 remainder 28
Solution to Exercise 65 (p. 12)
665 remainder 15
Solution to Exercise 67 (p. 12)
957 remainder 34
Solution to Exercise 69 (p. 12)
665 remainder 4
Solution to Exercise 71 (p. 13)
458 remainder 13
Solution to Exercise 73 (p. 13)
996 remainder 23
Solution to Exercise 75 (p. 13)
42
Solution to Exercise 77 (p. 13) 8,216

Solution to Exercise 79 (p. 13)
\$1,975 per month
Solution to Exercise 81 (p. 13)
$\$ 485$ each person invested
Solution to Exercise 83 (p. 13)
14 cubes per hour
Solution to Exercise 85 (p. 13)
8 bits in each byte
Solution to Exercise 87 (p. 14)
38
Solution to Exercise 89 (p. 14) 600,000
Solution to Exercise 91 (p. 14) 625,600


[^0]:    *Version 1.2: Aug 18, 2010 8:45 pm GMT-5
    ${ }^{\dagger}$ http://creativecommons.org/licenses/by/3.0/
    1 "Multiplication and Division of Whole Numbers: Concepts of Division of Whole Numbers" [http://cnx.org/content/m34864/latest/](http://cnx.org/content/m34864/latest/)

[^1]:    " "Addition and Subtraction of Whole Numbers: Whole Numbers" [http://cnx.org/content/m34795/latest/](http://cnx.org/content/m34795/latest/)
    4"Addition and Subtraction of Whole Numbers: Rounding Whole Numbers" [http://cnx.org/content/m34780/latest/](http://cnx.org/content/m34780/latest/)
    ${ }^{5 "}$ "Addition and Subtraction of Whole Numbers: Properties of Addition" [http://cnx.org/content/m34802/latest/](http://cnx.org/content/m34802/latest/)
    6 "Multiplication and Division of Whole Numbers: Multiplication of Whole Numbers"
    [http://cnx.org/content/m34863/latest/](http://cnx.org/content/m34863/latest/)
    ${ }^{7}$ "Multiplication and Division of Whole Numbers: Concepts of Division of Whole Numbers"
    [http://cnx.org/content/m34864/latest/](http://cnx.org/content/m34864/latest/)

